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On a Unified Theory of Electromagnetism and Weak Interaction

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Abstract

A unified theory of weak and electromagnetic interaction has been developed on the basis of the assumption that the charge and mass of a lepton is of dynamical origin. According to this model, the spontaneous breakdown of symmetry generates the photon as a Goldstone boson.

In recent times, a good deal of work has been accomplished on the development of a unified theory of electromagnetic and weak interactions (Weinberg, 1967). However, in doing so, we have generally come across the following difficulties:

(1) Electromagnetic interaction is mediated by photons but we do not know the physical existence of any such mediating particle in weak interaction.
(2) Quantum electrodynamics is renormalisable but weak interaction, in general, is not renormalisable.
(3) There is also the problem of symmetries and how they are broken in weak interactions. If we consider the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum, then we face the problem of the existence of Goldstone bosons. Weinberg (1967), however, avoided this difficulty by introducing the photon and the intermediate boson fields as gauge fields. Moreover, G. t. Hooft (1971) has shown that the theories of the type introduced by Weinberg are renormalisable. But still, we have some uncertainties regarding this theory. For the model predicts the existence of a neutral lepton current or the existence of heavy leptons. However, the present experimental evidence neither supports nor discards the theory at the moment.

In this context we may add here that a new theory of weak interactions has been developed (Bandyopadhyay, 1964, 1970, 1972) on the basis of the photon–neutrino weak coupling (Bandyopadhyay, 1968a, b) and dynamical origin of charge (Bandyopadhyay, 1965) where photons are taken to be fundamental field quanta. The interesting aspect of this theory is that it is renormalisable and there is no unitarity catastrophe at high energy (Bandyopadhyay, 1964, 1970, 1972). Indeed, it has been shown that in this theory the weak decay processes as well as the weak scattering processes in the low energy region and in the energy region near the threshold energy for baryonic resonances the results are identical with that derived from the current–current coupling theory (V–A theory), but in the high energy region the cross-section decreases with energy, the general behaviour being of the form $\sigma \propto 1/E_\nu$, where E_ν is the incident laboratory neutrino energy (Bandyopadhyay, 1964, 1970, 1972). Evidently there is no unitarity catastrophe in this theory. Also, as the basic interaction in this theory is the photon–neutrino weak coupling, it becomes renormalisable. In this note we shall show that the photon–neutrino weak coupling as well as the concept of dynamical origin of charge helps us to develop a unified theory of electromagnetism and weak interaction where the spontaneous breakdown of symmetry generates the photon as a Goldstone boson.

The concept of dynamical origin of charge in fact envisages a model of leptons where electron and muon are represented as $(\nu_e S_1)$ and $(\nu_\mu S_2)$ respectively, S_1 and S_2 being the systems of photons interacting 'weakly' with neutrinos. The interactions which give charge (as well as mass) to the neutrinos also add two more components transforming a two-component spinor to a four-component one (Bandyopadhyay, 1963). Indeed, in a previous paper (Bandyopadhyay, 1965) we have shown that though the neutrino theory of light suggests that photons can interact weakly (with neutrinos), and in local field theory this concept imposes a certain limitation on the 'electromagnetic character' of photons, yet in non-local field theory this is not effectively so, as we can get, in a certain limit, an 'electromagnetic effect' out of these weakly interacting fields if we consider n-photon interactions at different space-time points in the 'external' space. In this way we can get a geometrical representation of electric charge which gives the clue to the concept of dynamical origin of charge. For clarification we now recapitulate the mathematical details worked out in Bandyopadhyay (1965).

Let us consider the two-component spinor wave function $\psi(X, r)$ where X and r are external and internal space-time variables. It is considered that $\psi(X, r)$ satisfies the relation

$$\psi(X) = \int d^4 \gamma \psi(X, r) \tag{1}$$

It is further contended that the non-local spinor $\psi(X, r)$ obeys the Dirac equation in terms of the variable X

$$\left(\gamma^\mu \frac{\partial}{\partial X_\mu} + m\right) \psi(X, r) = 0 \tag{2}$$

The spinor current is expressed as

$$C^{\mu}(X) = \int d^4 r\, d^4 S \bar{\psi}(X, r) \gamma^{\mu} \psi(X, s) \tag{3}$$

Now assuming that the electromagnetic field quantity $A_{\mu}(r, t)$ also satisfies a similar relation

$$A_{\mu}(Y) = \int d^4 t A_{\mu}(Y, t) \tag{4}$$

we take n-photon fields at different space-time points in the external space as follows

$$A_{\mu}(Y - \tfrac{1}{2}\varepsilon_1) + A_{\mu}(Y + \tfrac{1}{2}\varepsilon_1) + A_{\mu}(Y - \tfrac{1}{2}\varepsilon_2) + A_{\mu}(Y + \tfrac{1}{2}\varepsilon_2) + \dots + A_{\mu}(Y - \tfrac{1}{2}\varepsilon_m) + A_{\mu}(Y + \tfrac{1}{2}\varepsilon_m) \tag{5}$$

From this we see that when $\varepsilon \to 0$, the expression just reduces to the single point potential given by $nA_{\mu}(Y)$ where $n = 2m$. Thus the interaction Lagrangian for n-photon weak interactions with the spinor takes the form

$$\begin{aligned} L_I &= ig\left[\sum_{i=1}^{m} C^{\mu}(X) A_{\mu}(Y - \tfrac{1}{2}\varepsilon_i) + \sum_{i=1}^{m} C^{\mu}(X) A_{\mu}(Y + \tfrac{1}{2}\varepsilon_i)\right] \\ &= ig \sum_{i=1}^{m} \int d^4 r\, d^4 s d^4 t[\bar{\psi}(X, r) \gamma^{\mu} \psi(X, s) A_{\mu}(Y_i, t) \\ &\quad + \bar{\psi}(X, r) \gamma^{\mu} \psi(X, s) A_{\mu}(\bar{Y}_i, t)] \end{aligned} \tag{6}$$

where $Y_i = Y - \frac{1}{2}\varepsilon_i$, $\bar{Y}_i = Y + \frac{1}{2}\varepsilon_i$ and g is the dimensionless weak coupling constant which is taken to have the value $g = 10^{-10}e$ (Bandyopadhyay, 1968a, b).

Now taking m such that $e/2m = g$, the weak coupling constant, we note that the system of interactions (6) in the limit $\varepsilon \to 0$ just reduces to the formal electromagnetic coupling

$$ie \int d^4 r\, d^4 s d^4 t \bar{\psi}(X, r) \gamma^{\mu} \psi(X, s) A_{\mu}(Y, t) \tag{7}$$

Thus in the limit $\varepsilon \to 0$, n-photon weak interactions can be considered to be 'equivalent' to the proper electromagnetic interaction (7) and by this a geometrical representation of e in terms of g is obtained.

We now show that the interaction (7) involving non-local fields which gives a geometrical description of electric charge is equivalent to a non-local interaction involving a form factor which can produce the mass of the lepton as well as add two more components to the spinor. Indeed, going over to the Fourier transform of the field quantities in equation (7), we write (Katayama, 1952)

$$\begin{aligned} L_I = \frac{ie}{(2\pi)^{12}} \int d^4 p_1\, d^4 p_2\, d^4 p_3\, E(p_1, p_2) \exp\left(-i \sum_i p_i V_i\right) \\ \bar{\psi}(-p_1) \gamma_{\mu} \psi(p_2) A_{\mu}(p_3) \end{aligned} \tag{8}$$

where

$$V_1 = V_2 = X, \qquad V_3 = Y$$

and

$$E(p_1,p_2) = \int d^4 r \exp\left(-\frac{ip_1 r}{2}\right) \rho_1(-p_1, r)$$
$$\times \int d^4 s \exp\left(-\frac{ip_2 s}{2}\right) \rho_2(p_2, s)$$
$$\times \int d^4 t \exp\left(-\frac{i(p_1 - p_2)t}{2}\right. \rho_3(-p_1 - p_2, t) \qquad (9)$$

Now assuming that the integral of the form

$$\int d^4 r\, e^{-r_1 q} \rho_i(-p, r) = F_i(p, q) \qquad (10)$$

depends only on the variable

$$\pi^2 = q^2 - \frac{(pq)^2}{p^2}$$

and $\lim F(\pi) = 1$, equation (9) can be written as

$$E(p_1,p_2) = F(p_1 + p_2, \tfrac{1}{2}(p_1 - p_2)) \qquad (11)$$

Substituting this in (7) and going back to the X-representation, we get the interaction in the form

$$L_I = ie \int d^4 X_1\, d^4 X_2\, d^4 X_3\, \bar{\psi}(X_1) \gamma_\mu \psi(X_2) F(X_1 - X_3, X_2 - X_3) A_\mu(X_3) \qquad (12)$$

This shows the equivalence between the two interactions.

To attain gauge invariance, the interaction Lagrangian is generally written as (Katayama *et al.*, 1959)

$$L_I = ie \int dx_1\, dx_2\, dx_3\, \bar{\psi}(x_1) \gamma_\alpha F(x_1 - x_3, x_2 - x_3)$$
$$\times \exp\left[-ie \int_{X_2}^{X_1} dx_\beta\, a_\beta(X)\right] \psi(x_2) A_\alpha(x_3) \qquad (13)$$

where the form factor is given by

$$F_{rs}(x_1 - x_3, x_2 - x_3) = \delta_{rs} S(x_1 - x_3, x_2 - x_3)$$
$$- \widehat{(rs)} \frac{\partial}{\partial x_{3\alpha}} V(x_1 - x_3, x_2 - x_3) \qquad (14)$$

r and s being spinor indices. S and V are now some scalar functions. For the requirement of gauge invariance, S is taken as

$$S(x_1 - x_3 - x_2 - x_3) = \delta^4(x_1 - x_3) \delta^4(x_2 - x_3) \qquad (15)$$

Thus the modified interaction becomes

$$\begin{aligned} L_I &= L_1 + L_2 \\ &= ie \int dx \bar{\psi}(x) \gamma_\alpha \psi(x) A_\alpha(x) \\ &\quad - \frac{ie}{2} \int dx_1\, dx_2\, dx_3\, \bar{\psi}(x_1) \gamma_\alpha \gamma_\beta \psi(x_2) F_{\alpha\beta}(x_3) \\ &\quad \times \exp\left[-ie \int_{x_2}^{x_1} dx_\gamma A_\gamma(x)\right] V(x_1 - x_3, x_2 - x_3) \end{aligned} \tag{16}$$

where $F_{\alpha\beta}$ is the electromagnetic force.

Now to show that this Lagrangian (16) can give rise to the whole mass of the spinor when the bare spinor is massless (Katayama *et al.*, 1959), we note that L_1 here is invariant under the transformation

$$\psi(x) \rightarrow \exp(i\beta\gamma_5)\psi(x) \tag{17}$$

showing that this term does not contribute to any mass effect. But the second term L_2 destroys the invariance under the transformation (17) showing that this term only can contribute to the mass effect.

Again, in a previous paper (Bandyopadhyay, 1963) we have shown that if we take ψ as a two-component spinor instead of a four-component one we can introduce internal freedom such that we can add two more components through the form factor of the interaction representing particle and anti-particle. Indeed, from equation (16) we find that the mass contributing factor L_2 changes its sign when we substitute

(i) $e \rightarrow -e$

(ii) $A_\mu(x) \rightarrow -A_\mu(x)$

(iii) $F(x_1 - x_3, x_2 - x_3) \rightarrow -F(x_1 - x_3, x_2 - x_3)$ (18)

Thus the form factor with positive and negative sign can provide us with the necessary internal freedom to accommodate the negative energy state (Bandyopadhyay, 1963). This picture of anti-particle evidently suggests that the structural details of particle and anti-particle are just mirror reflections of each other and this is in full conformity with the CP invariance.

From this analysis we observe that electron and muon can be described as $e^- = (\nu_e S)$, $\mu^- = (\nu_\mu S)$, where $\nu_e(\nu_\mu)$ is the bare massless and chargeless two-component spinor and S is the system of photons which, interacting weakly with $\nu_e(\nu_\mu)$, produces the charge as well as the mass of the electron (muon) and also adds two more components to the bare two-component spinor. According to the Lagrangian (16), the mass of the electron is given by (Katayama *et al.*, 1959)

$$m_e \simeq \frac{3\alpha}{8\pi} \frac{\delta}{me} \lambda^2 \ldots \tag{19}$$

where δ/m_e is a factor, the magnitude of which may be taken to be comparable with the deviation of the experimental value of the electron magnetic moment from the theoretical one and λ is a cut-off factor. Similarly the mass of the muon can be written as

$$m_\mu \simeq \frac{3\alpha}{8\pi}\frac{\delta}{m_\mu}\lambda'^2 \tag{20}$$

where δ/m_μ is a term similar to that of δ/me and λ' is the cut-off factor. Comparing this with equation (19) and from the observed mass of muon, we find $\lambda' \simeq 14{\cdot}35\lambda$. This change in the cut-off factor indicates the change in the form factor in equation (16) when ν_μ is taken to be the bare spinor instead of ν_e. Thus the muon–electron mass difference can be related to the existence of two kinds of neutrinos, ν_e and ν_μ, in nature. Also this picture of muon and electron explains in a nice way the muon–electron universality which consists of the fact that muons behave just like electrons in all interactions.

To show that this model of lepton helps us to unify weak and electromagnetic interactions, we now write the Lagrangian of electron and muon as follows

$$L = L_0 + L_{\text{e.m.}} + L_{\text{weak}} \tag{21}$$

where

$$L_0 = i\bar{\phi}_{\nu_e}\gamma_\mu\,\partial_\mu\,\phi_{\nu_e} + i\bar{\phi}_{\nu_\mu}\gamma_\mu\,\partial_\mu\,\phi_{\nu_\mu} + L_{\text{Maxwell}} \tag{21a}$$

$$L_{\text{e.m.}} = ig\sum^{n}\int dx_1\,dx_2\,dx_3\,\bar{\phi}_{\nu_e}(x_1)\,\gamma_\mu\,F(x_1 - x_2, x_1 - x_3)\left[\exp\int_{X_2}^{X_1} dx_\beta\,A_\beta(x)\right]$$
$$\phi_{\nu_e}(x_2)\,A_\mu(x_3) + \text{similar terms with } \phi_{\nu_\mu}\ldots \tag{21b}$$

and

$$L_{\text{weak}} = ig\bar{\phi}_{\nu_e}\gamma_\mu\,\phi_{\nu_e}\,A_\mu(S) + \text{similar terms for } \nu_\mu\ldots \tag{21c}$$

Here $\phi_{\nu_e}(\phi_{\nu_\mu})$ is the two-component wave function for $\nu_e(\nu_\mu)$ and $A_\mu(S)$ represents the potential corresponding to the system of photons. From equations (6), (7), (13), (14), (15) and (16), we note that by taking the limit for the n-photon fields such that the system of interactions $\sum^n$ reduces to a single photon interaction and with a specific choice of the form factor, equation (21b) reduces to the well-known electromagnetic interaction

$$L_{\text{e.m.}} = ie\bar{\psi}_e\,\gamma_\mu\,\psi_e\,A_\mu + \text{similar terms for } \mu\ldots \tag{21d}$$

Again, from equation (21c) we note that this leads effectively to the V–A form of weak interaction. Indeed, the point of interest here is that no phenomenological consideration is needed to have the coupling in the V–A form. For all our basic spinors are neutrinos and with two-component neutrinos the only bilinear we can construct is a vector $\bar{\phi}\gamma_\mu\phi$. This gives the V–A form when ϕ is written as $\frac{1}{2}(1 + \gamma_5)\psi$, where ψ is a four-component spinor. This scheme of weak interaction can be extended to the world of hadrons provided we consider a certain lepton–hadron relation (Bandyopadhyay, 1964, 1970, 1972). Evidently the Lagrangian L in equation (21) suggests that the symmetries relating the weak and electromagnetic interactions are exact symmetries of the Lagrangian but are broken by the vacuum. In fact, from equations (21a), (21b) and (21c), we note that the Lagrangian in equation (21) is invariant under the transformation $\phi \to e^{i\alpha\gamma_5}\phi$, we obtain self-consistent solutions of the theory which break the continuous γ_5 transformation invariance and generates the electromagnetic interaction (21d) by taking a specific choice for the form factor (as in equation (14)) and taking the limit for the n-photon fields such that the system of interactions $\sum^n$ reduces to a single photon interaction (as in equations (6) and (7)). This spontaneous breakdown of symmetry suggests the existence of a Goldstone boson and in our scheme we find the spontaneous breakdown of symmetry generates the photon as a Goldstone boson.

Finally we may add that the photon–neutrino weak coupling evidently suggests the existence of a neutral lepton current $\bar{\psi}_\nu \gamma_\mu (1+\gamma_5)\psi_\nu$. Thus we should have processes like $\nu_\mu + p \to \nu_\mu + p$ mediated by photon. Indeed, we have shown (Bandyopadhyay *et al.*, 1969) that the present theory gives a cross-section of the process such that

$$\frac{\sigma(\bar{\nu}_\mu + p \to \bar{\nu}_\mu + p)}{\sigma(\bar{\nu}_\mu + p \to n + e^+)} < \cdot 1 \tag{22}$$

which conforms well with the present experimental upper limit. Reines & Gurr (1970) are performing an experiment to find the cross-section for elastic scattering of electrons by fission anti-neutrinos. Comparing our predictions for the process (Bandyopadhyay & Raychaudhuri, 1971) with the present experimental upper limit as reported by Reines (1972), we find

$$\frac{\sigma(\text{exp})}{\sigma(\text{photo–neutrino coupling})} < 85\ldots \tag{23}$$

This is to be compared with the predictions of the conventional V–A theory and the Weinberg theory which give

$$\sigma(\text{exp}) < 1{\cdot}7\sigma(e\nu_e)(e\nu_e) \tag{24}$$

$$\sigma(\text{exp}) < 1{\cdot}3\sigma(\text{Weinberg}) \tag{25}$$

It may be added that if the experiments suggest that $\sigma(\text{exp}) < \frac{1}{4}\sigma(e\nu_e)(e\nu_e)$, then both the conventional current–current coupling and the Weinberg theory will be ruled out. Only future experiments will indicate the validity of any of these theories.

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